# Chapter 1

# Probability

## 1.1 Basic Concepts

**1.1-2** **(a)** $S = \{\text{bbb, gbb, bgb, bbg, bgg, gbg, ggb, ggg}\}$;

**(b)** $S = \{\text{female}, \text{male}\}$;

**(c)** $S = \{000, 001, 002, 003, \ldots, 999\}$.

**1.1-4** **(a)**

| Clutch size: | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency: | 3 | 5 | 7 | 27 | 26 | 37 | 8 | 2 | 0 | 1 | 1 |

**(b)**

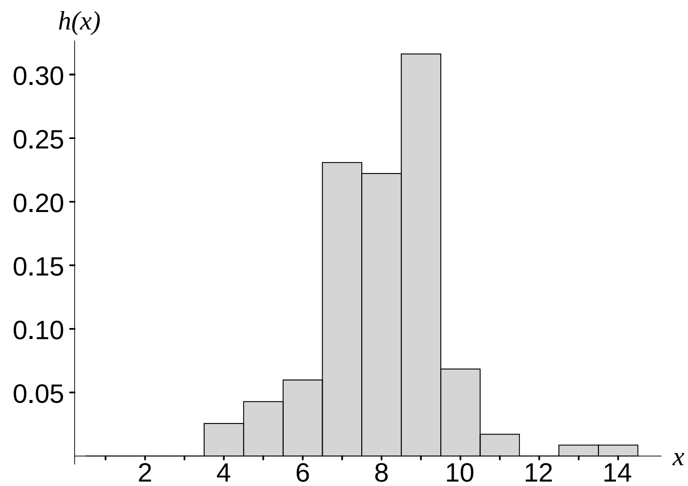


Figure 1.1–4: Clutch sizes for the common gallinule

**(c)** 9.

**1.1-6** **(a)**

| No. Boxes: | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 19 | 24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Frequency: | 10 | 19 | 13 | 8 | 13 | 7 | 9 | 5 | 2 | 4 | 4 | 2 | 2 | 1 | 1 |

**(b)**

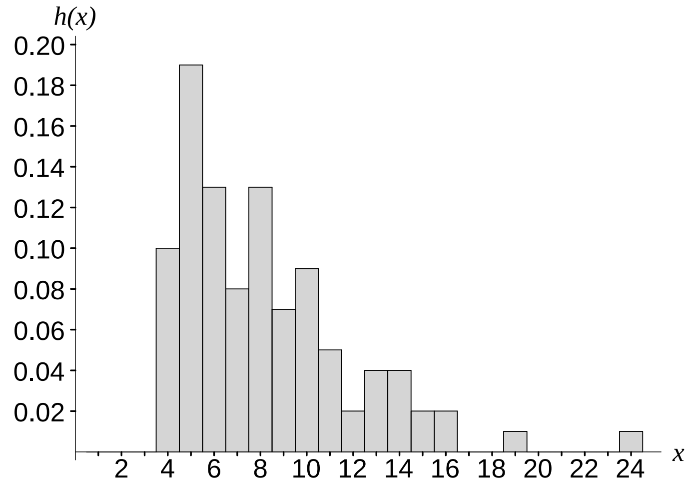


Figure 1.1–6: Number of boxes of cereal

**1.1-8** **(a)** $f(1) = \dfrac{2}{10}, f(2) = \dfrac{3}{10}, f(3) = \dfrac{3}{10}, f(4) = \dfrac{2}{10}.$

**1.1-10** This is an experiment.

**1.1-12** **(a)** $50/204 = 0.245$; $93/329 = 0.283$;

**(b)** $124/355 = 0.349$; $21/58 = 0.362$;

**(c)** $174/559 = 0.311$; $114/387 = 0.295$;

**(d)** Although James' batting average is higher that Hrbek's on both grass and artificial turf, Hrbek's is higher over all. Note the different numbers of at bats on grass and artificial turf and how this affects the batting averages.

## 1.2 Properties of Probability

**1.2-2** **(a)** $S = \{$HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTTH, TTHH, HTHT, THTH, THHT, HTTT, THTT, TTHT, TTTH, TTTT$\}$;

**(b)** **(i)** $5/16$, **(ii)** $0$, **(iii)** $11/16$, **(iv)** $4/16$, **(v)** $4/16$, **(vi)** $9/16$, **(vii)** $4/16$.

**1.2-4** **(a)** $1/4$;

**(b)** $P(B) = 1 - P(B') = 1 - P(A) = 3/4$;

**(c)** $P(A \cup B) = P(S) = 1$.

**1.2-6** **(a)** $P(A \cup B) = 0.4 + 0.5 - 0.3 = 0.6$;

**(b)**

$$\begin{aligned} A &= (A \cap B') \cup (A \cap B) \\ P(A) &= P(A \cap B') + P(A \cap B) \\ 0.4 &= P(A \cap B') + 0.3 \\ P(A \cap B) &= 0.1; \end{aligned}$$

**(c)** $P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.3 = 0.7$.

**1.2-8** **(a)**

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ 0.7 &= 0.4 + 0.5 - P(A \cap B) \\ P(A \cap B) &= 0.2; \end{aligned}$$

$$\textbf{(b)}\quad P(A' \cup B') = P[(A \cap B)'] = 1 - P(A \cap B) = 1 - 0.2 = 0.8.$$

**1.2-10**
$$\begin{aligned} A \cup B \cup C &= A \cup (B \cup C) \\ P(A \cup B \cup C) &= P(A) + P(B \cup C) - P[A \cap (B \cup C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)] \\ &= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) \\ &\quad + P(A \cap B \cap C). \end{aligned}$$

**1.2-12** **(a)** 1/3; **(b)** 2/3; **(c)** 0; **(d)** 1/2.

**1.2-14** **(a)** $S = \{(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)\}$;
**(b)** **(i)** 1/10; **(ii)** 5/10.

**1.2-16** $P(A) = \dfrac{2[r - r(\sqrt{3}/2)]}{2r} = 1 - \dfrac{\sqrt{3}}{2}$.

## 1.3 Methods of Enumeration

**1.3-2** $(4)(3)(2) = 24$.

**1.3-4** **(a)** $(4)(5)(2) = 40$; **(b)** $(2)(2)(2) = 8$.

**1.3-6** **(a)** $4\dbinom{6}{3} = 80$;

**(b)** $4(2^6) = 256$;

**(c)** $\dfrac{(4-1+3)!}{(4-1)!3!} = 20$.

**1.3-8** ${}_9P_4 = \dfrac{9!}{5!} = 3024$.

**1.3-10** $S =$ { FFF, FFRF, FRFF, RFFF, FFRRF, FRFRF, RFFRF, FRRFF, RFRFF, RRFFF, RRR, RRFR, RFRR, FRRR, RRFFR, RFRFR, FRRFR, RFFRR, FRFRR, FFRRR } so there are 20 possibilities.

**1.3-12** $3 \cdot 3 \cdot 2^{12} = 36,864$.

**1.3-14**
$$\begin{aligned} \binom{n-1}{r} + \binom{n-1}{r-1} &= \frac{(n-1)!}{r!(n-1-r)!} + \frac{(n-1)!}{(r-1)!(n-r)!} \\ &= \frac{(n-r)(n-1)! + r(n-1)!}{r!(n-r)!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}. \end{aligned}$$

**1.3-16**
$$0 = (1-1)^n = \sum_{r=0}^{n} \binom{n}{r} (-1)^r (1)^{n-r} = \sum_{r=0}^{n} (-1)^r \binom{n}{r}.$$

$$2^n = (1+1)^n = \sum_{r=0}^{n} \binom{n}{r} (1)^r (1)^{n-r} = \sum_{r=0}^{n} \binom{n}{r}.$$

**1.3-18**
$$\begin{aligned}\binom{n}{n_1, n_2, \ldots, n_s} &= \binom{n}{n_1}\binom{n-n_1}{n_2}\binom{n-n_1-n_2}{n_3}\cdots\binom{n-n_1-\cdots-n_{s-1}}{n_s} \\ &= \frac{n!}{n_1!(n-n_1)!}\cdot\frac{(n-n_1)!}{n_2!(n-n_1-n_2)!} \\ &\quad \cdot\frac{(n-n_1-n_2)!}{n_3!(n-n_1-n_2-n_3)!}\cdots\frac{(n-n_1-n_2-\cdots-n_{s-1})!}{n_s!0!} \\ &= \frac{n!}{n_1!n_2!\ldots n_s!}.\end{aligned}$$

**1.3-20** **(a)** $\dfrac{\binom{19}{3}\binom{52-19}{6}}{\binom{52}{9}} = \dfrac{102,486}{351,325} = 0.2917;$

**(b)** $\dfrac{\binom{19}{3}\binom{10}{2}\binom{7}{1}\binom{3}{0}\binom{5}{1}\binom{2}{0}\binom{6}{2}}{\binom{52}{9}} = \dfrac{7,695}{1,236,664} = 0.00622.$

## 1.4 Conditional Probability

**1.4-2** **(a)** $\dfrac{1041}{1456}$;

**(b)** $\dfrac{392}{633}$;

**(c)** $\dfrac{649}{823}$.

**(d)** The proportion of women who favor a gun law is greater than the proportion of men who favor a gun law.

**1.4-4** **(a)** $P(\text{HH}) = \dfrac{13}{52}\cdot\dfrac{12}{51} = \dfrac{1}{17}$;

**(b)** $P(\text{HC}) = \dfrac{13}{52}\cdot\dfrac{13}{51} = \dfrac{13}{204}$;

**(c)** $P$(Non-Ace Heart, Ace) + $P$(Ace of Hearts, Non-Heart Ace)

$$= \frac{12}{52}\cdot\frac{4}{51} + \frac{1}{52}\cdot\frac{3}{51} = \frac{51}{52\cdot 51} = \frac{1}{52}.$$

**1.4-6** Let $A = \{3 \text{ or } 4 \text{ kings}\}$, $B = \{2, 3, \text{ or } 4 \text{ kings}\}$.

$$\begin{aligned}P(A|B) &= \frac{P(A\cap B)}{P(B)} = \frac{N(A)}{N(B)} \\ &= \frac{\binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}}{\binom{4}{2}\binom{48}{11} + \binom{4}{3}\binom{48}{10} + \binom{4}{4}\binom{48}{9}} = 0.170.\end{aligned}$$

**1.4-8** **(a)** $\dfrac{8}{14}\cdot\dfrac{7}{13} = \dfrac{56}{182}$;

**(b)** $\dfrac{6}{14} \cdot \dfrac{5}{13} = \dfrac{30}{182}$;

**(c)** $2\left(\dfrac{8}{14} \cdot \dfrac{6}{13}\right) = \dfrac{96}{182}$ or $1 - \left[\dfrac{56}{182} + \dfrac{30}{182}\right] = \dfrac{96}{182}$.

**1.4-10** **(a)** Let $A = \{$2 WIN and 4 LOSE in first 6 selections$\}$, B = $\{$WIN on 7th selection$\}$.

$$\begin{aligned} P(A \cap B) &= P(A) \cdot P(B|A) \\ &= \frac{\binom{3}{2}\binom{17}{4}}{\binom{20}{6}} \cdot \frac{1}{14} = \frac{1}{76} = 0.01316; \end{aligned}$$

**(b)** $\displaystyle\sum_{k=1}^{9} \frac{\binom{3}{2}\binom{17}{2k-2}}{\binom{20}{2k}} \cdot \frac{1}{20-2k} = \frac{35}{76} = 0.4605.$

**1.4-12** $\dfrac{\binom{2}{0}\binom{8}{5}}{\binom{10}{5}} \cdot \dfrac{2}{5} + \dfrac{\binom{2}{1}\binom{8}{4}}{\binom{10}{5}} \cdot \dfrac{1}{5} = \dfrac{1}{5}.$

**1.4-14** **(a)** $P(A) = \dfrac{52}{52} \cdot \dfrac{51}{52} \cdot \dfrac{50}{52} \cdot \dfrac{49}{52} \cdot \dfrac{48}{52} \cdot \dfrac{47}{52} = \dfrac{8,808,975}{11,881,376} = 0.74141$;

**(b)** $P(A') = 1 - P(A) = 0.25859$.

**1.4-16** **(a)** It doesn't matter because $P(B_1) = \dfrac{1}{18},\ P(B_5) = \dfrac{1}{18},\ P(B_{18}) = \dfrac{1}{18}$;

**(b)** $P(B) = \dfrac{2}{18} = \dfrac{1}{9}$ on each draw.

**1.4-18** $\dfrac{3}{5} \cdot \dfrac{5}{8} + \dfrac{2}{5} \cdot \dfrac{4}{8} = \dfrac{23}{40}$.

**1.4-20** **(a)** $P(A_1) = 30/100$;

**(b)** $P(A_3 \cap B_2) = 9/100$;

**(c)** $P(A_2 \cup B_3) = 41/100 + 28/100 - 9/100 = 60/100$;

**(d)** $P(A_1 \,|\, B_2) = 11/41$;

**(e)** $P(B_1 \,|\, A_3) = 13/29$.

## 1.5 Independent Events

**1.5-2** **(a)**

$$\begin{aligned} P(A \cap B) &= P(A)P(B) = (0.3)(0.6) = 0.18; \\ P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.6 - 0.18 \\ &= 0.72. \end{aligned}$$

**(b)** $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{0.6} = 0.$

**1.5-4** Proof of **(b)**:
$$\begin{aligned} P(A' \cap B) &= P(B)P(A'|B) \\ &= P(B)[1 - P(A|B)] \\ &= P(B)[1 - P(A)] \\ &= P(B)P(A'). \end{aligned}$$

Proof of **(c)**:
$$\begin{aligned} P(A' \cap B') &= P[(A \cup B)'] \\ &= 1 - P(A \cup B) \\ &= 1 - P(A) - P(B) + P(A \cap B) \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A')P(B'). \end{aligned}$$

**1.5-6**
$$\begin{aligned} P[A \cap (B \cap C)] &= P[A \cap B \cap C] \\ &= P(A)P(B)P(C) \\ &= P(A)P(B \cap C). \end{aligned}$$

$$\begin{aligned} P[A \cap (B \cup C)] &= P[(A \cap B) \cup (A \cap C)] \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C). \end{aligned}$$

$$\begin{aligned} P[A' \cap (B \cap C')] &= P(A' \cap C' \cap B) \\ &= P(B)[P(A' \cap C') \,|\, B] \\ &= P(B)[1 - P(A \cup C \,|\, B)] \\ &= P(B)[1 - P(A \cup C)] \\ &= P(B)P[(A \cup C)'] \\ &= P(B)P(A' \cap C') \\ &= P(B)P(A')P(C') \\ &= P(A')P(B)P(C') \\ &= P(A')P(B \cap C') \end{aligned}$$

$$\begin{aligned} P[A' \cap B' \cap C'] &= P[(A \cup B \cup C)'] \\ &= 1 - P(A \cup B \cup C) \\ &= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + \\ &\qquad P(B)P(C) - PA)P(B)P(C) \\ &= [1 - P(A)][1 - P(B)][1 - P(C)] \\ &= P(A')P(B')P(C'). \end{aligned}$$

**1.5-8** $\dfrac{1}{6} \cdot \dfrac{2}{6} \cdot \dfrac{3}{6} + \dfrac{1}{6} \cdot \dfrac{4}{6} \cdot \dfrac{3}{6} + \dfrac{5}{6} \cdot \dfrac{2}{6} \cdot \dfrac{3}{6} = \dfrac{2}{9}.$

**1.5-10** **(a)** $\dfrac{3}{4} \cdot \dfrac{3}{4} = \dfrac{9}{16};$

**(b)** $\dfrac{1}{4} \cdot \dfrac{3}{4} + \dfrac{3}{4} \cdot \dfrac{2}{4} = \dfrac{9}{16};$

**(c)** $\dfrac{2}{4} \cdot \dfrac{1}{4} + \dfrac{2}{4} \cdot \dfrac{4}{4} = \dfrac{10}{16}.$

**1.5-12** **(a)** $\left(\dfrac{1}{2}\right)^3 \left(\dfrac{1}{2}\right)^2;$

**(b)** $\left(\dfrac{1}{2}\right)^3 \left(\dfrac{1}{2}\right)^2;$

**(c)** $\left(\dfrac{1}{2}\right)^3 \left(\dfrac{1}{2}\right)^2;$

**(d)** $\dfrac{5!}{3!\,2!}\left(\dfrac{1}{2}\right)^3\left(\dfrac{1}{2}\right)^2$.

**1.5-14** **(a)** $1-(0.4)^3 = 1-0.064 = 0.936$;

**(b)** $1-(0.4)^8 = 1-0.00065536 = 0.99934464$.

**1.5-16** **(a)** $\displaystyle\sum_{k=0}^{\infty}\frac{1}{5}\left(\frac{4}{5}\right)^{2k} = \frac{5}{9}$;

**(b)** $\dfrac{1}{5}+\dfrac{4}{5}\cdot\dfrac{3}{4}\cdot\dfrac{1}{3}+\dfrac{4}{5}\cdot\dfrac{3}{4}\cdot\dfrac{2}{3}\cdot\dfrac{1}{2}\cdot\dfrac{1}{1} = \dfrac{3}{5}$.

**1.5-18** **(a)** 7; **(b)** $(1/2)^7$; **(c)** 63; **(d)** No! $(1/2)^{63} = 1/9,223,372,036,854,775,808$.

**1.5-20**

| n | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| **(a)** | 0.7037 | 0.6651 | 0.6536 | 0.6480 | 0.6447 |
| **(b)** | 0.6667 | 0.6319 | 0.6321 | 0.6321 | 0.6321 |

**(c)** Very little when $n > 15$, sampling with replacement
Very little when $n > 10$, sampling without replacement.
**(d)** Convergence is faster when sampling with replacement.

## 1.6 Bayes's Theorem

**1.6-2** **(a)**
$$\begin{aligned} P(G) &= P(A\cap G)+P(B\cap G)\\ &= P(A)P(G\,|\,A)+P(B)P(G\,|\,B)\\ &= (0.40)(0.85)+(0.60)(0.75) = 0.79; \end{aligned}$$

**(b)**
$$\begin{aligned} P(A\,|\,G) &= \frac{P(A\cap G)}{P(G)}\\ &= \frac{(0.40)(0.85)}{0.79} = 0.43. \end{aligned}$$

**1.6-4** Let event $B$ denote an accident and let $A_1$ be the event that age of the driver is 16–25. Then

$$\begin{aligned} P(A_1\,|\,B) &= \frac{(0.1)(0.05)}{(0.1)(0.05)+(0.55)(0.02)+(0.20)(0.03)+(0.15)(0.04)}\\ &= \frac{50}{50+110+60+60} = \frac{50}{280} = 0.179. \end{aligned}$$

**1.6-6** Let $B$ be the event that the policyholder dies. Let $A_1, A_2, A_3$ be the events that the deceased is standard, preferred and ultra-preferred, respectively. Then

$$\begin{aligned} P(A_1\,|\,B) &= \frac{(0.60)(0.01)}{(0.60)(0.01)+(0.30)(0.008)+(0.10)(0.007)}\\ &= \frac{60}{60+24+7} = \frac{60}{91} = 0.659;\\ P(A_2\,|\,B) &= \frac{24}{91} = 0.264;\\ P(A_3\,|\,B) &= \frac{7}{91} = 0.077. \end{aligned}$$

**1.6-8** Let $A$ be the event that the VCR is under warranty.

$$\begin{aligned} P(B_1 \mid A) &= \frac{(0.40)(0.10)}{(0.40)(0.10) + (0.30)(0.05) + (0.20)(0.03) + (0.10)(0.02)} \\ &= \frac{40}{40 + 15 + 6 + 2} = \frac{40}{63} = 0.635; \\ P(B_2 \mid A) &= \frac{15}{63} = 0.238; \\ P(B_3 \mid A) &= \frac{6}{63} = 0.095; \\ P(B_4 \mid A) &= \frac{2}{63} = 0.032. \end{aligned}$$

**1.6-10** **(a)** $P(AD) = (0.02)(0.92) + (0.98)(0.05) = 0.0184 + 0.0490 = 0.0674;$

**(b)** $P(N \mid AD) = \dfrac{0.0490}{0.0674} = 0.727;\ P(A \mid AD) = \dfrac{0.0184}{0.0674} = 0.273;$

**(c)** $P(N \mid ND) = \dfrac{(0.98)(0.95)}{(0.02)(0.08) + (0.98)(0.95)} = \dfrac{9310}{16 + 9310} = 0.998;$

$P(A \mid ND) = 0.002.$

**(d)** Yes, particularly those in part (b).